THERMOFLUIDS - Continuum Model, Basic Equations, Momentum Diffusion.

THE CONTINUUM MODEL
* Fluids made up of too many molecules to follow individually.
* Consider average properties at a point in space e.g $\underline{v}$ field: $\underline{v}(x,y,z)$
* Look at molecular motion against a piston: $p = F/A = \dot{p}/A$
  $\rightarrow \Delta p$ on each collision $\propto m v_i$, frequency per unit area $\propto n_v v_i$
  $\rightarrow p = \dot{P}/A = m n_v v_i^2 \times const. = (n_v m) \times (2 \times const./m) \times (\overline{m v_i^2}/2) = \rho R T$
  $\rightarrow$ Similarly other macroscopic properties like μ, λ, c etc. from this motion.
* Knudsen Number, $Kn = \lambda/L = $ mean free path / characteristic space dimension.
  $\rightarrow$ Continuum works only when averaging over many molecules and collisions so breakdown occurs at $Kn = 1$
* When considering fields must use partial derivatives, ∇ etc.

CONSERVATION OF MASS, EULER, BERNOULLI EQUATIONS
* Take a control volume $\delta x \delta y \delta z$, consider net flux; $\frac{\delta M}{\delta t} = -\left(\frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y}\right)\delta x \delta y \delta z$
  $\rightarrow$ but $M = \rho \delta x \delta y \delta z$, as $\delta t \rightarrow 0 \Rightarrow \frac{\partial \rho}{\partial t} = -\nabla \cdot \rho \underline{v}$
* Material Derivative $\frac{D}{Dt} \equiv \left(\frac{\delta}{\delta t} + \underline{v} \cdot \nabla\right)$, follows an element of fluid.
  $\rightarrow$ Comprises steady and unsteady terms. (steady due to motion thro' field)
* Consider forces on $\delta x \delta y \delta z$ cube, then $\underline{f} = m\underline{a}$ gives $-\nabla p = \rho \frac{d\underline{v}}{dt}$
  $\rightarrow$ Note here $\underline{v}$ is held by fluid element. Use material derivative to find the Euler equation in terms of field properties: $-\nabla p = \rho \left[\frac{\partial \underline{v}}{\partial t} + (\underline{v} \cdot \nabla)\underline{v}\right]$
* Now apply Euler along a streamline in steady flow
  $\rightarrow \rho v \frac{\partial v}{\partial n}\hat{e}_n = -\hat{e}_n \frac{\partial p}{\partial n} - \hat{e}_n \frac{\partial p}{\partial y}^{0} \Rightarrow \frac{\partial}{\partial n}\left(p + \frac{1}{2}\rho v^2\right) = 0, \quad p + \frac{1}{2}\rho v^2 = const.$
  $\rightarrow$ If no vorticity $(\nabla \times \underline{v})$ every streamline has same Bernoulli constant.
  $\rightarrow$ In streamline coordinates, $v\frac{\partial v}{\partial s} = \frac{-1}{\rho}\frac{\partial p}{\partial s}$ and $\frac{v^2}{R} = \frac{1}{\rho}\frac{\partial p}{\partial n}$

NO SLIP CONDITION AND MOMENTUM TRANSFER
* Fluid molecules stick to surfaces for long enough to reach thermal equilibrium
  $\rightarrow$ So after collision have same average $\underline{v}$ and T as surface
* Molecules which have just left surface collide into nearby molecules.
  $\rightarrow$ Repeated collisions transfer momentum deeper into fluid
  $\rightarrow$ Momentum diffusion results in linear velocity profile.
* Adjacent layers of fluid exchange momentum at rate $\propto \frac{\partial v_x}{\partial y}$
  $\rightarrow$ But rate of change of momentum across an area is a force
  $\rightarrow \tau = F/A = \mu \frac{\partial v_x}{\partial y}$ where constant of proportionality μ is viscosity.
  $\rightarrow$ In gases μ increases with T, average molecular speed increases
  $\rightarrow$ In liquids μ decreases with T, bond energy becomes less than KE
* Newtonian Fluid — momentum diffusion rate proportional to velocity gradient or strain rate. Shear thickening/thinning are Non-Newtonian.

NAVIER-STOKES EQUATION - $\underline{f} = m\underline{a}$ for viscous fluid.
* Similar derivation to Euler, include extra viscous force terms
* In vector notation: $\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v} = \frac{-1}{\rho}\nabla p + \frac{\mu}{\rho}\nabla^2 \underline{v}$

NON-EXAMINABLE

THERMOFLUIDS — Flow Between Plates, Boundary Layers

FLOWS BETWEEN PLATES — Couette and Poiseuille

* Couette: steady, incompressible flow between plates, relative velocity V
  → No acceleration, forces on a control volume sum to zero.
  → x-dirn ↔ $\frac{\partial \tau}{\partial y} \delta y \delta x - \frac{\partial p}{\partial x} \delta x \delta y = 0$, but $\frac{\partial p}{\partial x} = 0 \Rightarrow \frac{\partial \tau}{\partial y} = 0$
  → Now substitute $\tau = \mu \frac{\partial v_x}{\partial y} \Rightarrow \mu \frac{\partial^2 v_x}{\partial y^2} = 0 \Rightarrow v_x = \frac{Vy}{D}$ with boundary conds.

* Poiseuille: stationary plates with pressure gradient
  → Same control volume analysis $\Rightarrow \frac{\partial \tau}{\partial y} = \frac{\partial p}{\partial x}, \quad \mu \frac{\partial^2 v_x}{\partial y^2} = \frac{\partial p}{\partial x}$
  → Solve to get $v_x = \left(\frac{1}{2\mu} \frac{\partial p}{\partial x}\right)\left(y^2 - \frac{D^2}{4}\right)$ with boundary conds.

* Can do same force balance for viscous flow down slope, etc.
* For combined Couette and Poiseuille, boundary conditions different.
  → favourable $\frac{\partial p}{\partial x} \Rightarrow$ parabolic v-profile. Steeper gradient at stationary plate.
  → adverse $\frac{\partial p}{\partial x} \Rightarrow$ parabolic again, velocity reverses near stationary plate.
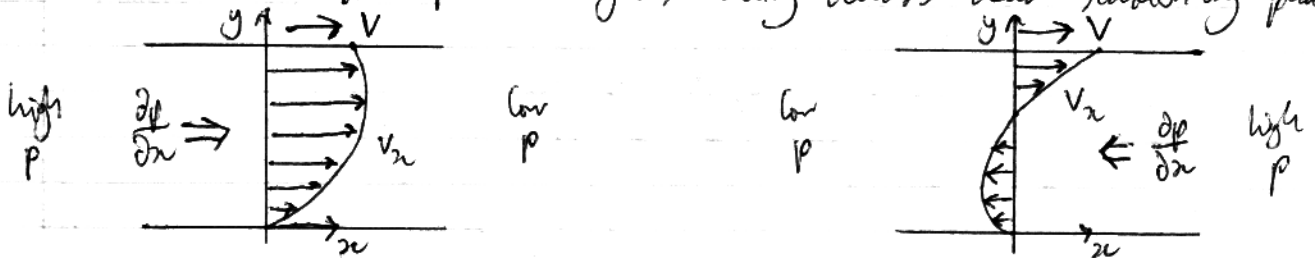
BOUNDARY LAYERS — region where viscous dominate over inertial forces

* Consider single plate in unbounded fluid. Fluid next to plate takes plate velocity by no slip condition.
  → This layer exerts shear stresses on next layer up
  → Continues as momentum diffuses away from plate.
  → Faster with more viscous fluids.
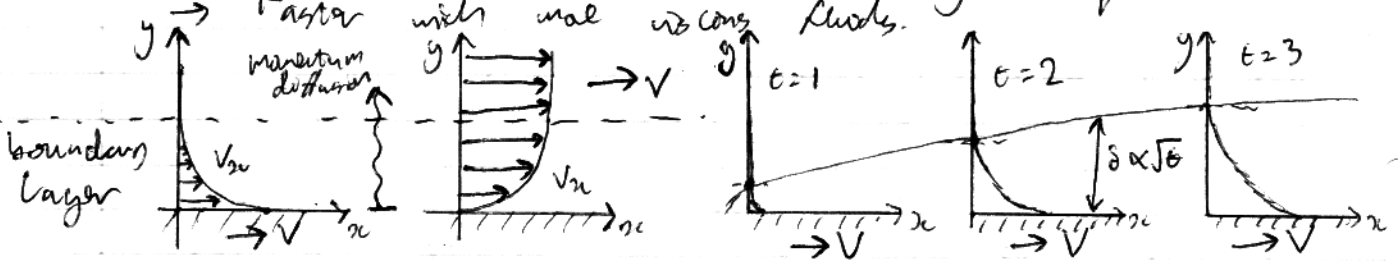
BOUNDARY LAYER GROWTH — by diffusion (kinematic viscosity)

* For moving plate, dimensional analysis on $\delta(\nu, t)$ gives $\underline{\delta \sim \sqrt{\nu t}}$
* For moving fluid, dim. an. on $\delta(\nu, x, V) \Rightarrow \underline{\delta \sim \sqrt{\frac{\nu x}{V}}}$, x ∂ distance along plate.

BERNOULLI IN A BOUNDARY LAYER

* Inside viscous forces dominate $\Rightarrow$ Bernoulli invalid
* However, v_y changes very little in x-direction, streamlines almost parallel.
  → p in boundary layer ≈ free stream pressure above it.

THERMOFLUIDS — Turbulent, Separating Boundary Layers

BOUNDARY LAYERS WITH PRESSURE GRADIENT
* If free stream accelerating, must be favourable pressure gradient.
  → Velocity gradient at wall becomes steeper.
* If free stream decelerating must have adverse pressure gradient.
  → Velocity gradient at wall becomes less steep, can reverse.

favourable $\frac{\partial p}{\partial x}$ $\frac{\partial p}{\partial x} = 0$ increasing adverse p-gradient ⟹
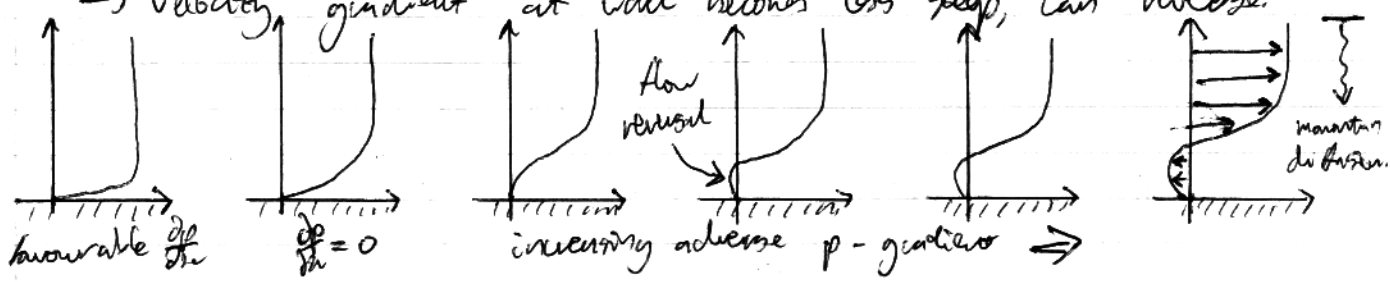
BOUNDARY LAYER SEPARATION
* First consider pressure field of inviscid flow.
  → High pressure regions at stagnation points, lower elsewhere.
* But real fluids form a boundary layer, growing from front to back.
* The pressure field generates adverse pressure gradients
  → Can cause flow reversal — fluid must <u>separate</u> from surface.

DELAYING SEPARATION — Good because separation increases drag.
* Increase momentum diffusion to win over adverse pressure gradient.
* Greater viscosity increases momentum transfer, delays separation.
* Can inject high momentum fluid into boundary layer
  → Or suck low momentum fluid from flow — impractical.

MOMENTUM LOSS IN BOUNDARY LAYER

* Con. of mass: $\rho V (h+\Delta-\delta) + \int_0^\delta \rho v\, dy = \rho V h$

* force on CV = net momentum flux: $-\int_0^x \tau_w\, dx = \rho V^2 (h+\Delta-\delta) + \int_0^\delta \rho v^2 dy - \rho V^2 h$

$$\Rightarrow \int_0^x \tau_w\, dx = V \int_0^\delta \rho v\, dy - \int_0^\delta \rho v^2 dy$$

* But velocity profile in laminar boundary layer $v(y) \approx V \left[\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \right]$
  → Combine to get $\underline{\delta/x = 4.64/\sqrt{Re_x}}$

BOUNDARY LAYER TRANSITION TO TURBULENCE
* At low Re viscous forces strong enough to damp out disturbances
* At high Re non-linear, unstable inertial effects tend to dominate
  → Above a critical Re, Tollmien-Schlichting waves grow, cause turbulence.
* Note, always a laminar sub-layer very close to wall.
* Turbulence increases momentum transfer rate ⟹ more gradual, fuller velocity profile and increased skin friction.
  → Also increases transfer from free stream ⟹ resistant to separation.
* Often occurs when about to separate, velocity inflexion point unstable.
* Layers can sometimes separate, become turbulent, then re-attach due to increased momentum transfer.

THERMOFLUIDS — Dimensional Analysis, Compressible Flow

DIMENSIONAL ANALYSIS
* Buckingham Pi Law: Variables − Dimensions = Independent Groups.
* Dimensionless plots independent of Sarkozy! General procedure:
1) List dependent and relevant independent variables
2) Apply Buckingham Pi Law, generate dimensionless numbers (see D.B.)
3) May have to split effect into multiple independent components
4) Match dimensionless numbers for complete similarity.

COMPRESSIBLE FLOW — $p = \rho RT$ not. ρ = const.
* Also need to find energy field to solve problems
* Remember SFEE. In D.B. u = KE of disordered motion,
$v^2/2$ = KE of ordered motion, gz = GPE, p/ρ = pressure potential energy.
* Stagnation enthalpy $h_0 = h + \frac{1}{2}v^2$, if fluid slows to $V=0$ without heat or work.
→ $h_0 + gz$ is constant if no heat or work transfer.
↳ stagnation $T = h_0/c_p$

ENTROPY — $s = k \ln \Omega$
* Entropy increases as internal energy increases.
* In an adiabatic process, S can never decrease.
* Viscous dissipation: KE of ordered motion decreases, KE of disordered motion increases so temperature also increases.
* In a flow process, KE of ordered motion transferred to enthalpy
→ But h_0 remains constant. Net result is drop in mechanical energy, increase in thermal energy and an increase in system disorder.
→ This increase in disorder measured by ΔS, irreversible process
* Entropy measures distribution between thermal and mechanical energy.
→ Low entropy ⇒ more mechanical energy ⇒ more useful work potential.

THERMAL TO MECHANICAL ENERGY TRANSFER
* For incompressible flows, no way to reverse viscous dissipation
→ i.e. Thermal field can't affect mechanical field.
* However for compressible flows, we can heat a parcel to change ρ
→ The fluid expands and does work on surrounding fluid!
→ So the equation of state $p = \rho RT$ allows this conversion.

INCOMPRESSIBLE ASSUMPTION
* Consider a perfect gas with changing velocity
→ If no heat or work transfer h_0 = const, $dh + vdV = 0 \Rightarrow c_p dT + V dV$
→ Without viscous dissipation, isentropic $T\rho^{(1-\gamma)}$ = const. $\Rightarrow \frac{dT}{T} + (1-\gamma)\frac{d\rho}{\rho} = 0$
→ Substitute, use $c_p(\gamma - 1) = \gamma R$, $\frac{d\rho}{\rho} = -\left(\frac{v^2}{\gamma RT}\right)\frac{dv}{dv} = -M^2 \frac{dv}{v}$

* So in a perfect gas, density change is very small for low Mach numbers with $M = \frac{v^2}{\gamma RT} = \frac{V}{a}$, a = speed of sound

THERMOFLUIDS — Pipe Flow, Total Pressure

LAMINAR FLOW IN CIRCULAR PIPE

* Friction coefficient, c_f, is wall stress non-dimensionalised with dynamic pressure
* In fully developed flow, no momentum build up, so pressure drop across length must balance wall shear stress.
  $$\rightarrow -2\pi R \tau_w - \pi R^2 \frac{\partial p}{\partial x} = 0 \quad \Rightarrow \quad c_f = -\frac{R}{\rho V^2}\frac{\partial p}{\partial x}$$
* In a pipe, boundary layers grow from entrance, eventually meet.
* To find velocity profile, balance forces on cylindrical elements
  $$\rightarrow \mu \frac{dv_x}{dr} = \frac{r}{2}\frac{\partial p}{\partial x}, \text{ integrate with bound. cond.} \Rightarrow V_x = \frac{-\partial p}{\partial x}\left(\frac{R^2 - r^2}{4\mu}\right)$$
  $$\rightarrow \text{So average velocity } V = \frac{-R^2}{8\mu}\frac{\partial p}{\partial x}$$
  $$\rightarrow \text{Substitute for } \frac{\partial p}{\partial x} \text{ in } c_f \text{ expression} \Rightarrow c_f = \frac{8\mu}{\rho V R} = \frac{16}{Re}$$
* So c_f depends only on Re for laminar pipe flow.

LAMINAR VS. TURBULENT FLOW, MIXING

* Above a certain Re c_f independent of Re
  → Small perturbations unstable, grow rapidly ⇒ unsteady turbulence.
  → However, time-averaged quantities do have steady values.
* Molecular diffusion is only transport process between laminar layers
* In turbulent flow, fluid moves between layers in large-scale eddies
  → So mixing rate and momentum transport increases dramatically.
  → Higher momentum transfer ⇒ higher τ_w ⇒ greater pressure drop.
  → Velocity profile not parabolic — flatter near centre.
* Could model eddies by increasing μ, but sizes vary with position, upstream flow and additional μ varies throughout the fluid!
* Roughness defined as ratio of average bump height to pipe diameter
  → Rough pipes create large eddies, increase pressure drop.

STATIC, STAGNATION AND TOTAL PRESSURE — Losses in 1D pipes.

* Static p — measured with no change of velocity
* Stagnation p — measured at stagnation point, flow has come to rest.
* Total p — includes height term: $\underline{p_T = p + \rho V^2/2 + \rho g h}$
  → Conserved if no loss of mechanical energy in flow.
  → Rises as flow goes through pumps, fall through other components.
* General loss proportional to dynamic pressure. $\Delta p_0 = \frac{1}{2}\rho V^2 K$
  → K calculated/plotted for orifice plates, bends, expansions etc.
  → Along a pipe, must remember $\underline{K = fL/D}$ with $f = 4c_f$
* Note p_0 is mechanical energy per unit volume.
  → so for a pump $\Delta p_0 = -\dot{w}_x \left(\frac{\rho}{\dot{m}}\right)$

NETWORK ANALYSIS

* In large tanks, assume $V = 0$
  → So can use conservation of p_T between surfaces of tanks
  (because surface is at p_{atm})
* Generally, follow total pressure changes along different paths in network, solve simultaneously.

THERMOFLUIDS - Lift, Drag and Wakes

LIFT AND DRAG - the important resultant forces
* Non dimensionalise with dynamic pressure and frontal/wing area.
  → If incompressible flow $C_D = f$ (Shape, Re, angle of attack, roughness)
* Drag has two components
  → Skin friction, τ_w, due to boundary layers
  → Form drag where flow separates, makes low-pressure wake.

FLOW AT V. LOW RE - creeping flow
* Fluid moving slowly ⇒ inertial forces ≫ viscous forces
  → Neglect inertial terms in Navier-Stokes equation ⇒ $\nabla p \approx \mu \nabla^2 \underline{v}$
  → This can be solved analytically for a sphere.
  → It turns out $C_D = \dfrac{24}{Re}$
* No separation, all drag due to skin friction
* Pressure gradient pushes fluid round in flow direction.

FLOW AT LOW RE - for a sphere $Re \approx 100$
* Boundary layer separates just behind shoulder
* Low pressure recirculating region forms ⇒ form drag
* As Re increases further, point of separation moves forward.
* Beyond $Re = 1000$, skin friction is negligible

FLOW AT HIGH RE - for a sphere $Re > 200000$
* Boundary layer becomes turbulent before it reaches equator
  → This increases momentum transfer, delays separation
  → Size of wake reduces, reduction in form drag; suddenly.

DRAG REDUCTION
* Many important applications are at high Re, form drag ≫ skin friction
* Reduction in form drag achieved by delaying boundary layer separation.
  → Adverse pressure gradients should be as gentle as possible.
  → However has side effect of increasing skin friction.
* Can inject high momentum air or suck out low momentum air
* Or trigger turbulence, e.g. by roughening body surface.

FLOW INSTABILITY AND VORTEX SHEDDING
* Separating boundary layers create shear layers
  → Unstable because of inflexion points in shear velocity profiles.
  → Develop waves which then roll up into vortices.
* The two parallel shear layers behind a bluff body even more unstable.
  → Shed vortices alternately from either side of cylinder.

THERMOFLUIDS – Conductive Heat Transfer

CONDUCTIVE HEAT TRANSFER – Energy transfer down temperature gradient
* Fourier's Law of Heat Conduction: $\dot{Q}/A = \dot{q} = -\lambda \frac{\partial T}{\partial x}$
  → Constant of proportionality - thermal conductivity λ W/mK
* Consider steady state, 1D heat flow along a slab between T_1 and T_2
  → For an element thickness dx, at steady state no accumulation
  → Heat fluxes must balance: $\dot{q}_x = \dot{q}_{x+dx} \Rightarrow -\lambda \frac{\partial T}{\partial x} = -\lambda \frac{\partial T}{\partial x} + \frac{\partial}{\partial x}(\dot{q}_x) dx$
  → So $\frac{\partial}{\partial x}\left(-\lambda \frac{\partial T}{\partial x}\right) = 0 \Rightarrow$ if λ constant, temperature gradient and $\dot{q}$ constant.
* Integrate this, apply B.Cs: $T = T_1 - \frac{x}{L}(T_1 - T_2)$ and $\underline{\dot{Q} = A\lambda(T_1 - T_2)/L}$
  → Can write $\boxed{\dot{Q} = \Delta T / R_{Th}}$ with $\underline{R_{Th} = L/\lambda A}$ by Ohm's law analogy

ADVANCED CONDUCTION – Steady
* Two different slabs in series, between T_1 and T_3, interface at T_2
  → $R_A \dot{Q} = (T_1 - T_2)$ $R_B \dot{Q} = (T_2 - T_3)$ for each slab
  → $\dot{Q}$ is uniform, add to get $\dot{Q}(R_A + R_B) = T_1 - T_3$
  → i.e. heat transfer resistances in series add.
* For convective heat transfer to a flowing fluid from wall
  → Heat flux $\propto \Delta T$: $\underline{\dot{Q} = -hA(T_\infty - T_{wall})}$, $\underline{R_{Th} = 1/hA}$
* Circular pipe; T_1 at r_1 inner surface; T_2 at r_2 outer surface
  → Consider a thin ring → $\dot{Q} = -2\pi r L \lambda \frac{\partial T}{\partial r}$
  → $\dot{Q} = $ const. $\dot{q} = \dot{Q}/A$ varies with r $\dfrac{2\pi L \lambda (T_1 - T_2)}{\ln(r_2/r_1)}$
  → Integrate between surfaces to get $\dot{Q} =$
  → So $\underline{R_{Th} = \ln(r_2/r_1)/2\pi L \lambda}$ for cylinder

UNSTEADY HEAT CONDUCTION – relax steady state assumption
* Consider same slab as before: accumulation = in − out + created
  → internal heat generation G W/m³, look at δT in δt, specific heat c
  → $\rho c A dx \frac{\delta T}{\delta t} = \dot{q}_x A - \left[\dot{q}_x + \frac{\partial \dot{q}_x}{\partial x} dx\right] A + G A dx$
  → if λ is constant this reduces to $\underline{\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c}\frac{\partial^2 T}{\partial x^2} + \frac{G}{\rho c}}$ (usually $G=0$)
  → $\lambda/\rho c = \alpha = $ thermal diffusivity
* Conduction in a semi-infinite slab – error function solution
  → Surface held at T_s, $T = T_0$ everywhere in slab initially
  → Then $T = T_0 - (T_s - T_0) \, \text{erf}\left(x/2\sqrt{\alpha t}\right)$, where x is depth coordinate
  → Characteristic time for heat to penetrate to $x \sim t = x^2/\alpha$
  → More generally can use $\tau = s^2/\alpha = (V/A)^2/\alpha$, heated up when $\underline{F_0 = \frac{\alpha \tau}{s^2} = 1}$

LUMPED HEAT CAPACITY ANALYSIS – Assume body at uniform T
* If valid, can write $c\rho V \frac{\partial T}{\partial t} = -hA(T - T_\infty)$
* Solution $T = T_\infty + (T_0 - T_\infty) \exp(t/\tau_c)$ where $\tau_c = \dfrac{c\rho V}{hA}$
* Valid if internal $R_{Th} \ll$ surface R_{Th}, i.e. $\left(\frac{s}{\lambda A}\right) \ll \left(\frac{1}{hA}\right)$, $\frac{hs}{\lambda} \ll 1$
  → Define $Bi = \frac{hs}{\lambda}$, valid if $\underline{Bi < 0.1}$ say.

THERMOFLUIDS – Heat Exchangers and Forced Convection

HEAT EXCHANGERS – Co-flow analysis

* Define $\dot{Q} = \Delta T / R_{Th} = UA\Delta T$ – $U = 1/R_{Th}A$ = overall heat transfer coefficient.
* Consider a parallel flow heat exchanger

$\rightarrow$ For element dA, $d\dot{Q} = -\dot{m}_h c_h dT_h = \dot{m}_c c_c dT_c$
  but $d\dot{Q} = U dA (T_h - T_c)$ also. $\quad \hookrightarrow d(T_h - T_c) = -d\dot{Q}\left(\frac{1}{\dot{m}_h c_h} + \frac{1}{\dot{m}_c c_c}\right)$

$\rightarrow$ Then substitute for $d\dot{Q}$: $\quad \dfrac{d(T_h - T_c)}{T_h - T_c} = -U\left(\frac{1}{\dot{m}_h c_h} + \frac{1}{\dot{m}_c c_c}\right)dA$

$\rightarrow$ Integrate from one end to other, apply $\dot{Q} = \dot{m}_h c_h (T_{h1} - T_{h2}) = \dot{m}_c c_c (T_{c2} - T_{c1})$

$\rightarrow$ Finally, $\underline{\dot{Q} = UA\Delta T_m}$ with $\underline{\Delta T_m = (\Delta T_2 - \Delta T_1)/\ln(\Delta T_2 - \Delta T_1)}$

* Co-flow has an intermediate limiting T, in counter flow one stream approaches the inlet T of the other.
* Max possible heat exchange $\underline{\dot{Q}_{max} = (\dot{m}c)_{min} \Delta T_{max}}$
* Effectiveness $\underline{\varepsilon = \dot{Q}/\dot{Q}_{max}}$

FORCED CONVECTION – estimating values for h

* Incompressibility means thermal doesn't effect mechanical field.
 $\rightarrow$ Requires small Mach number, so $c_p T \gg v^2/2$, SFEE becomes $\dot{Q} = mc\Delta T$
* To evaluate material properties use $T_{film} = (T_W + T_\infty)/2$
* Diffusion of ordered momentum $\rightarrow$ shear stress due to velocity gradient.
* Diffusion of disordered motion $\rightarrow$ heat flux due to temperature gradient.
 $\rightarrow$ Pr = momentum diffusivity/thermal diffusivity = ν/α
 $\rightarrow$ Reynolds' Analogy – $Pr = 1$. Non dimensional heat and mass transfer equations are identical. $\underline{(T - T_W)/(T_\infty - T_W) = v_x/V}$, $\underline{St = c_f/2}$

HEAT TRANSFER COEFFICIENTS

* Nusselt Number – Normalise by characteristic conductive heat flux
 $\rightarrow$ Nu = heat flux/heat flux with no flow = $h\Delta T / (\lambda \frac{\Delta T}{D}) = \underline{hD/\lambda}$
* Stanton Number – Normalise by characteristic convection heat flux
 $\rightarrow$ St = heat flux/convection heat flux = $h\Delta T / \rho V c_p \Delta T = \underline{h/\rho V c_p}$
* If $Pr \neq 1$, have a momentum and a thermal boundary layer.
 $\rightarrow$ Relative thickness determined by Pr
 $\rightarrow$ Empirical correlations give St or Nu, e.g. $\boxed{St = \frac{1}{2}c_f Pr^{-\frac{2}{3}}}$
* Reynolds' Analogy still applies in turbulent layers
 $\rightarrow$ Eddies increase both momentum and thermal transfer.
* For general problems use dimensional analysis, $\underline{Nu_{av} = \dfrac{h_{av}D}{\lambda} = f_n(Re, Pr)}$

THERMOFLUIDS — Natural Convection and Radiation

CHARACTERISTICS OF NATURAL CONVECTION

* When heated, fluids expands and are driven upwards by buoyancy.
  → No natural reference velocity, no natural Re for flow.
* Define coefficient of expansion $\beta = \frac{1}{v}\frac{\partial v}{\partial T}\big|_p = \frac{-1}{\rho}\frac{\partial \rho}{\partial T}\big|_p$ because $\rho v = 1$
* So for small density changes, $\boxed{\delta\rho = -\rho\beta\,\delta T}$
* Liquids have β in tables, can derive $\beta = \frac{1}{T}$ from perfect gas eqn. of state.
* Considering natural convection from a flate plate, and assuming balance between buoyancy and viscous forces dominates, $\mu\frac{\partial^2 v}{\partial y^2} \approx \rho\beta(T-T_\infty)g$
  → Scaling analysis gives $V_{scale} = \rho\beta g\Delta T\delta^2/\mu$, a reference velocity!
* Define Grasshof Number $Gr = \rho V_{scale} D/\mu = \underline{\beta g\Delta T D^3/\nu^2}$
* Velocities low ⇒ Boundary layers laminar.
  → No separation because low pressure gradients assumed.
* In general dimensional analysis, remember $\beta\Delta T g$ must appear together.
  → of $\dot{Q} = fn(D, \rho, \mu, \lambda, c_p, \beta\Delta T g) \Rightarrow \underline{Nu = fn(Gr, Pr)}$

BLACK BODIES — Perfect emitters and absorbers

* Max rate of emission $E_b = \sigma T^4$ (Stefan - Boltzmann)
* For real surfaces, emissivity $= E/E_b = \varepsilon$, $\underline{E = \varepsilon\sigma T^4}$
  → A 'grey body' has ε independent of wavelength.
* When radiation strikes a surface, fraction ρ reflected, fraction α absorbed and fraction τ transmitted. $\underline{\rho + \alpha + \tau = 1}$, $\tau \doteq 0$ usually.
* Due to quantum effects, $\varepsilon = \alpha$, can be proved for a black enclosure.

RADIATION NETWORKS — Radiation to/from non-black bodies.

* Define G = Irradiation = Total incident radiation (W/m²)
* Define J = Radiosity = Total radiation leaving surface (W/m²)
  → Total leaving radiation has black body and reflected components: $\underline{J = \varepsilon E_b + \rho G}$
  → Because $\rho = 1 - \alpha = 1 - \varepsilon$, $G = (J - \varepsilon E_b)/(1-\varepsilon)$
  → But $\dot{Q}/A = J - G$, substitute for $G \Rightarrow \dot{Q} = \dfrac{E_b - J}{(1-\varepsilon)/\varepsilon A} = E_b - J/R$
  → Another 'Ohm's Law', with $R = \dfrac{1-\varepsilon}{\varepsilon A}$
* For radiation between surfaces, define $F_{12} = \dfrac{E\ \text{leaving }(1)}{E\ \text{arriving at }(2)}$
  → EM rays travel in straight lines, $\Rightarrow A_1 F_{12} = A_2 F_{21}$, reciprocity relationship.
* Net exchange $\dot{Q}_{12} = J_1 A_1 F_{12} - J_2 A_2 F_{21}$ but by reciprocity $\dot{Q}_{12} = J_1 - J_2/(1/A_1 F_{12})$
  → So 'space resistance' $\underline{R = 1/A_1 F_{12}}$
* Note, shape factors add up to unity, convex surfaces have self shape factor $= 0$
* When object in large enclosure, net radiation exchange independent of G
  → $\dot{Q} = A_1 \varepsilon_1 (\sigma T_1^4 - \sigma T_2^4)$, shape as if enclosure black body

RADIATION IN THE ENVIRONMENT

* Without the greenhouse effect, life on earth wouldn't exist
    Blah, Blah, Blah . . .

THERMOFLUIDS - Steady Flow Analysis, Availability, Gas Turbines

STEADY FLOW DEVICE ANALYSIS

* Usually one exit, one inlet. KE usually small
  → 1st Law: $\dot{Q} - \dot{W}_x = \dot{m}(h_e - h_i)$, neglecting $v^2/2$ and gz terms.
    Enthalpy of flow, determines 'quantity' of energy.
  → 2nd Law: $\dot{m}(s_e - s_i) = \int d\dot{Q}/T + \Delta\dot{S}_{irrev}$
    Entropy of flow, determines energy 'quality' and work potential.
* Irreversible Entropy created at inefficiencies. Mechanisms:
  → Viscous dissipation in boundary layers ⎤
  → Viscous mixing of different velocity flows ⎬ increase disorder
  → Heat transfer across a finite ΔT ⎦
* In general irreversible entropy generated when differences in properties
  even out by themselves without doing work.
* For a flow of fixed total energy, entropy determines energy
  not available for potential work.

MAX AVAILABLE POWER - 2nd Law Analysis

* Consider removing $d\dot{Q}$ at T from a flow, what is max available $d\dot{W}_Q$?
  → To get more power drive reversible heat engine, reject to environment T_0
  → 1st Law on heat engine ⇒ $d\dot{W}_Q = d\dot{Q} - d\dot{Q}_0$, 2nd Law ⇒ $\frac{d\dot{Q}}{T} - \frac{d\dot{Q}_0}{T_0} = 0$
  → Substitute to get $d\dot{W}_Q = \left(1 - \frac{T_0}{T}\right)d\dot{Q}$
* So as $T \to \infty$ this $\dot{Q}$ is as good as shaft work $\dot{W}_Q$
  → But as $T \to T_0$ becomes useless, $\dot{W}_Q = 0$
* Define specific steady flow availability function. $\boxed{b = h - T_0 s}$
  → By considering above situation, with control volume around device and heat engine
    we find maximum power extractable per kg as decrease in b.
* Dead state - when fluid in equilibrium with environment has b_0
  → exergy is max power that can be extracted from a fluid in a
    given state $e = b - b_0$
* Irreversibilities cause reduction in power potential $\underline{T_0 \Delta\dot{S}_{irrev}}$

GAS TURBINES - Joule Cycle is ideal and reversible

* Main 'real' effect is irreversible entropy creation in boundary layers
* Define isentropic efficiency $\eta_T = \dot{W}_{real}/\dot{W}_{ideal}$, $\eta_C = \dot{W}_{ideal}/\dot{W}_{real}$
  → Can find all states using isentropic relations and given η_T, η_C
* $\eta_{1st} = \eta_{thermal} = \underline{\dot{W}_{net}/\dot{Q}_{in}}$, $\eta_{2nd} = \eta_{rational} = \underline{\dot{W}_{net}/\dot{b}_{in}}$

* Jet engines: Turbine Work = Compressor Work
  → Exhaust into isentropic nozzle, KE is large at nozzle exit.
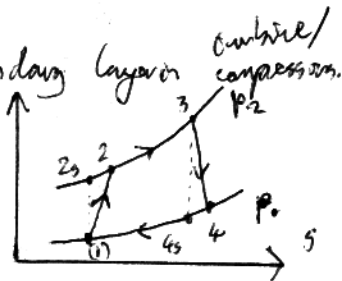
THERMOFLUIDS - Working Fluids, Thermodynamic Properties, Humidity

WORKING FLUIDS - Other than perfect gases.
* E.g. Air, Water, HFCs etc. Medium for heat and work transfer.
* Real fluids permit cycles closer to Carnot, heat addition at nearer const. T.
* Recall that two properties fix thermodynamic state.
  → We experimentally determine $p-V-T$ surfaces for substances.
  → Saturation state - phase change starts and ends
  → Vapour dome - region of two-phase liquid vapour states
  → Critical point - saturated liquid and vapour lines meet
  → Critical temperature - T at critical point, max T where liquid and vapour coexist
  → Triple line - line along which three phases coexist.

FINDING THERMODYNAMIC PROPERTIES - Databook Tables or Graphs.
* Dryness fraction: $x = m_g/m$, $m = m_f + m_g$, $V = m_f v_f + m_g v_g$
  → So $v = V/m = v_f + x v_{fg} = \underline{(1-x) v_f + x v_g}$ (same rule applies to $h, u,$ etc.)
* Tables in D.B. for perfect, semi-perfect to look up c_p, c_v, h etc.
* D.B. Tables for water and steam have saturation values for v_f, v_g etc.
* May have to use linear interpolation: $y = y_1 + \dfrac{(y_2 - y_1)}{(x_2 - x_1)} (x - x_1)$
* Charts in D.B. may be quicker.

PROPERTIES OF GAS MIXTURES
* Dalton's Law: mixture pressure is sum of partial pressures in same V, T
* Amagat's Law: mixture volume is sum of partial volumes, at same p, T
  → For an ideal gas, equation of state $\Rightarrow \underline{p_i = p(n_i/n)}$
* For extensive properties like U, H, S, $\underline{U = \sum m_i u_i}$ with $u = \sum \dfrac{m_i}{m} u_i$

MIXTURES OF GASES AND VAPOURS - e.g. dry air, water vapour, water
* If liquid in volume will evaporate until air becomes saturated
  → p_v rises as more water evaporates, until reaches $p_{v,sat}$.
* Specific humidity or humidity ratio $\omega = m_v/m_a = \left(\dfrac{M_v}{M_a}\right)\left(\dfrac{p_v}{p - p_v}\right)$
  → Using ideal gas equation of state, $\omega = M_v p_v / M_a p_a = \left(\dfrac{M_v}{M_a}\right)\left(\dfrac{p_v}{p - p_v}\right)$
* Relative humidity $\phi = n_v/n_{v,sat}$
  → Again equation of state gives $\underline{\phi = \dfrac{p_v}{p_{v,sat}}}$
* Dew point: $\phi = 1$, where water vapour begins to condense.

THERMOFLUIDS — Rankine Cycles

RANKINE CYCLE — Used in Steam Turbines

* Can't practically use a Carnot Cycle with steam
   → Whole cycle in two-phase region — expansion/compression difficult.
   → Critical point limits maximum temperature of heat addition ∴ efficiency.

(1) * Rankine Cycle — Pump in single phase region, take heat from lower T
   → But still operate turbine in two-phase region - droplets erode blades

(2) * Superheated Rankine Cycle — Boiler heats onto vapour region
   → Turbine in less of the two phase region. X near 1 so v. small droplets.
   → Average temperature of heat input rises, efficiency rises.

* Work in compressor small compared to turbine, sometimes neglected.
* Water virtually incompressible, $V \approx const.$ so $-w_c = V \Delta p$ in pump.
* Isentropic efficiencies $\eta_T = \dot{W}_{real} / \dot{W}_{ideal}$, $\eta_c = \dot{W}_{ideal} / \dot{W}_{real}$ can be used to account for irreversibilities.

IMPROVING EFFICIENCY OF RANKINE CYCLE

* In general raise av. T at which Q added.
   → Or lower av. T at which Q rejected.

(3) * Lower condenser pressure — T_{av} of Q_{out} drops
   → But if lower than p_{atm} could leak, turbine more two-phase.

(4) * Superheat more — T_{av} of Q_{in} rises, turbine less two-phase
   → But max T limited by metallurgical considerations.

(5) * Increase boiler pressure — T_{av} of Q_{in} rises
   → But peak T limited by materials, turbine operates more in vapour phase.

(6) * Expand then reheat — Lowers moisture content at turbine exit.
   → Raises T_{av} of Q_{in}. Benefit of each added reheat halves.

* Combined Gas-Vapour Cycle — steam turbine takes gas turbine exhaust heat.
   → Need HRSG. Consider distance-temperature diagram. Minimum
      temperature difference, pinch point, where water becomes saturated.
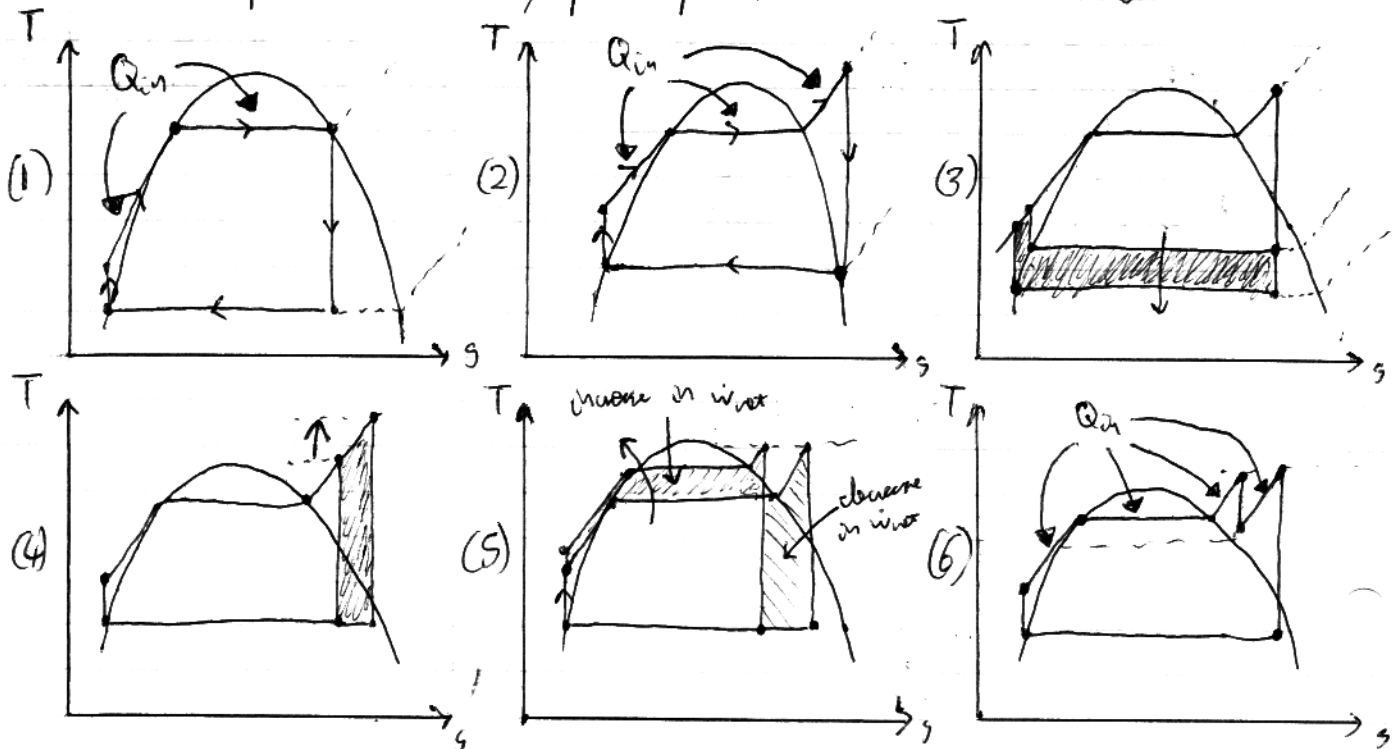
THERMOFLUIDS — Refrigeration and Combustion

REFRIGERATION CYCLES — Move heat from cold to hot

* Won't happen spontaneously by 2nd Law. $\Delta S_{tot} = \dfrac{\Delta Q_c + W}{T_h} - \dfrac{\Delta Q_c}{T_c} = 0$
  → Best is reversible fridge with
* Fridge: $COP_R = Q_c/W$, Heat Pump: $COP_{HP} = Q_h/W = 1 + COP_R$

(1) * Ideal Cycle — Reversed Carnot — $COP_R = T_c / T_h - T_c$

(2) * Real Cycle — Compress in single-phase vapour region, replace turbine with a throttle valve, for simplicity and cost.
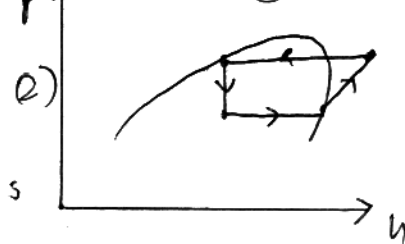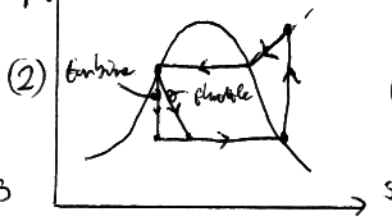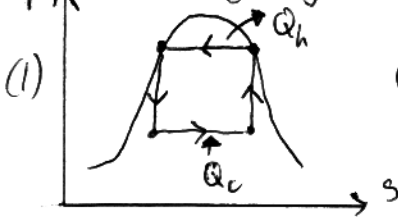
* Refrigerants must be cheap, stable, inert, non-toxic
  → Δp corresponding to operating T range small to minimise compressor work
  → vapour pressure low to reduce condenser cost, above pin or stop leakage.
  → latent heat of evaporation high to keep mass flow low.
* Gas only cycles without phase change achieve very low T

(1) (2) (2)

CHEMICAL EQUATIONS, LAMBDA (Assume air 21% O_2, 79% N_2)

* Stoichiometry: First place fuel-derived products on RHS. Then use O on RHS to set air on LHS. Nitrogen carried over to RHS.
* $AFR = \dot{m}_a / \dot{m}_f = n_a M_a / n_f M_f$. Excess air adds to both sides of equation.
* Lambda, $\lambda = AFR / AFR_{stoich}$. $\lambda > 1$ lean ie excess air, $\lambda < 1 \Rightarrow$ rich, $\phi = 1/\lambda$
* Sometimes have to solve simultaneous eqns to get chemical equation.
* Usually assume H_2O is water vapour — wet basis
  → Calculate molar fraction to get partial pressure
  → Then if dew point temperature < combustion temperature, assumption valid.

1st LAW APPLIED TO COMBUSTION

* Split into three processes to find T after reaction.
  1) Remove heat to bring from initial state to 25°C
  2) Remove heat while combusting, T stays constant at 25°C
  3) Add heat to raise to final temperature. = (1) + (2)
* Can look up values for (2) in databook
* Be very careful with units, per mol vs. per kg quantities, etc.