Project GA2: Turboexpander Demonstrator Notes

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Abstract

These notes are a record of the author's experience of demonstrating the Turboexpander project. The basic required theory and common issues are discussed on a per-week basis, and suggestions for innovative measurements are given.

Week 1: Suction box test

In order to select the design operating point for the turboexpander, the pressure rise characteristic must be measured. The required data are gauge pressures from the tube intake and exit, p_1 and p_2, and atmospheric conditions. Mass flow is given by applying Bernoulli's equation through the intake,

$$\dot{m} = \rho_\infty A_1 \sqrt{2(p_\infty - p_1)/\rho_\infty} \ , \tag{1}$$

and total-to-total pressure rise is,

$$\Delta p_{0\mathrm{sb}} = p_{0\infty} - p_{02} \ = p_{01} - (p_2 + \frac{1}{2}\rho_\infty V_2^2) = p_1 - p_2 \ , \tag{2}$$

using $p_{0\infty} = p_{01}$ and $V_1 = V_2$. Power is the product of stagnation pressure rise and volume flow rate,

$$\dot{W}_{\mathrm{xsb}} = \dot{m}\Delta p_{0\mathrm{sb}}/\rho_\infty \tag{3}$$

The mechanical efficiency is derived from a run-down test. The moment of inertia of the assembly is approximately $I = 2500\,\mathrm{kg\,mm^{-2}}$. Then the friction torque is,

$$\mathscr{T} = -I\frac{\mathrm{d}\omega}{\mathrm{d}t} \ . \tag{4}$$

The bearing and windage torque may be estimated analytically at of order $25\,\mathrm{N\,mm}$ each, the sum of which should correspond to the measured value.

Finally the design point is set. Guess $\omega = 8000\,\mathrm{rpm}$, and component efficiencies $\eta_\mathrm{t} = 0.65$, $\eta_\mathrm{c} = 0.6$. The design mass flow and pressure rise corresponding to the measured maximum power point, $\dot{m}^*$ and $\Delta p^*_{0\mathrm{sb}}$. Using the friction torque $\mathscr{T}$, and an arbitrary initial guess for mechanical efficiency η_m, solve iteratively Equations (5),

$$\eta_\mathrm{ov} = \eta_\mathrm{m}\eta_\mathrm{c}\eta_\mathrm{t} \ , \tag{5a}$$

$$\dot{W}_\mathrm{xt} = \frac{\dot{m}^*\eta_\mathrm{t}\Delta p^*_{0\mathrm{sb}}}{\rho_\infty(1-\eta_\mathrm{ov})} \ , \tag{5b}$$

$$\eta_\mathrm{m} = 1 - \mathscr{T}\omega/\dot{W}_\mathrm{xt} \ . \tag{5c}$$

Once the iteration is converged, calculate the quantities needed to set the velocity triangles and design the turbomachinery,

$$\dot{W}_{\mathrm{xc}} = \eta_{\mathrm{m}} \dot{W}_{\mathrm{xt}} \ , \tag{6a}$$

$$\Delta p_{0\mathrm{c}} = \Delta p^{*}_{0\mathrm{sb}} \frac{\eta_{\mathrm{ov}}}{1-\eta_{\mathrm{ov}}} \ , \tag{6b}$$

$$\Delta p_{0\mathrm{t}} = \Delta p^{*}_{0\mathrm{sb}} \frac{1}{1-\eta_{\mathrm{ov}}} \ . \tag{6c}$$

Week 2: Blade design

Compressor

The shaft speed is set by the compressor, in particular, the choice of rotor exit angle β_2 or amount of 'backsweep'. Use the slip factor $\sigma = 0.85$ to account for deviation. Combining with continuity and Euler's work equation, assuming no inlet swirl, gives a quadratic to solve for the rotational speed at a chosen β_2. A solution must be found that limits the relative velocity ratio across the rotor to 1/3, a diffuser chart estimate.

With the rotational speed fixed, determine the rotor inlet angle β_1 for 5° negative incidence compared to α_1^{rel}. The number of blades is estimated from the requirement that the pressure surface velocity is greater than zero, using an approximate mid-radius analysis. Allow a 25% safety factor; $6 \leq N \leq 20$ are sensible values. We can go back and use the number of blades to calculate a new slip factor $\sigma(N)$

There is considerable freedom to set the blade shape between fixed inlet and outlet angles β_1 and β_2. A range of shapes can be defined by integrating the following equation,

$$\frac{\mathrm{d}}{\mathrm{d}r} \left(\tan \beta(r) \right) = f(r) \ . \tag{7}$$

The choice of $f(r) = kr$, where k is a constant, gives radially-uniform loading. Quadratic or other variations can also be specified for $f(r)$. To plot out the blade shape in standard polar coordinates, we need $r(\theta)$ which is found by numerically integrating up from r_1 and an arbitrary datum θ position,

$$\tan \beta(r) = r \frac{\mathrm{d}\theta}{\mathrm{d}r} \quad \Rightarrow \quad \delta\theta = \frac{\tan \beta(r)}{r} \delta r \ . \tag{8}$$

Once this is done, the throat area between adjacent blades can be computed. Some adjustment of $f(r)$ may be needed to ensure this is consistent with the desired mass flow.

Angular momentum is approximately conserved across the gap (sized chosen by the designer) between rotor and stator, yielding stator inlet flow conditions. Allow for zero incidence on to the stators, setting β_3. Design the stators using diffuser charts, about 10° divergence after throat; the outlet angle is not very important.

Turbine

The turbine rotor inlet tangential velocity is set by applying Euler's work equation accros the rotor. We require no rotor exit swirl, and an allowance must be made for mechanical loss between the compressor and turbine,

$$r_5 V_{\theta 5} = r_2 V_{\theta 2} / \eta_{\mathrm{m}} \ . \tag{9}$$

The slip occurring in the compressor acts in reverse in the turbine, and must be accounted for to get the tangential velocity. Continuity gives the radial velocity and hence rotor inlet metal angle.

To achieve no swirl at the rotor exit, slip must be accounted for, bur there are no suitable correlations available. Set the metal angle 5° more negative than the relative flow angle. The throat area is more important, and should match closely the relative velocities at the throat and turbine exit,

$$Nt = 2\pi r_7 \cos \alpha_7^{\mathrm{rel}} \,. \tag{10}$$

The number of rotor blades may be chosen using the same mid-radius analysis as for the compressor stator, allowing a 25% safety factor. The number of blades is sensitive to the compressor designer's choice of β_2, and if a very large number are predicted to be needed, some compromise is required to reduce the rotational speed and turbine loading to sensible levels. Detailed shapes may be found using Equations (7) and (8).

The stator is more straightforward to desigh as it is highly accelerating. Leave a gap of about 5% local radius between rotor and stator. The throat area should match the velocities at throat and exit, i.e. create no diffusion between throat and trailing edge. The highest curvature should be at the throat, where the sides become parallel, with a slight reversal downstream. Select the number of blades based on a pitch to chord ratio of 0.75 as a start, but a multiple of the number of transfer ports yields more uniform inlet conditions.

Week 3: Manufacture and test

- Can print a scale drawing of the blades and bend by hand, or use clamping formers laser cut or 3D-printed, or 3D-print an entire ring on a backing plate.
- Do not be concerned with aerodynamic loss due to large glue fillets: blade loss is more detrimental to the performance of the machine.
- Make sure that static pressure tap tubes are flush with the surface, and not blocked.
- A new hole should be drilled to measure the compressor exit pressure.
- Remove all shims from behind the rotors before deciding that your blades are too tall and need filing down.
- Take data at a low speed before the blades fail, centrifugal loading is proportional to the angular velocity squared.
- Measurements require to calculate the objective function:
  - Intake static pressure, water manometer, to determine mass flow;
  - Compressor exit pressure, mercury manometer, to split up the pressure drops across the two components;
  - Turbine exit pressure, mercury manometer, to determine total loss.
- Additional measurements:
  - Radial pressure distributions;
  - Temperatures to derive component efficiencies;
  - Flow visualisation using tufts or paint.

Week 4: Modifications

- Change tip clearance using shims behind rotor disc.
- Swap components from different builds of the same turbine to quantify manufacturing variations.
- New stator blades if a problem has been identified.
- Run the compressor with a vaneless diffuser.
- Seal all holes in the cover and wrap the machine in duct tape to eliminate leakage.
- Leave the turbine stator throat unglued to allow geometry adjustments.