Module 3A5: Power Generation
Notes on Examples Paper 2

James Brind
jb753@cam.ac.uk

Michaelmas Term 2017

Abstract

This document outlines the methods used to answer each question on the second Power Generation examples paper. The results are commented on, and extra information is included for interest. This material is intended to complement the more detailed worked solutions available in the official cribs. Statements of physical principles and definitions are highlighted in **bold**, while assumptions are *emphasised*; we are told to treat air as a *perfect gas* throughout.

Q1: Steam turbine with reheat

This question is largely Part I material–calculating the states around the steam cycle using the **First Law** and definitions of **isentropic efficiency**. If we are only interested in efficiency, then neglecting feed pump work is a good approximation. However, the absolute feed pump power is still several megawatts, and if we are constructing a power station we have to know how big a pump to buy. For an *isentropic process*, $\mathrm{d}s = 0$, and setting this in the **fundamental equation** gives $\mathrm{d}h = \mathrm{d}p/\rho$. For an incompressible fluid such as water, $\rho \approx$ const. within the feed pump. Then applying the **First Law** and integrating from the inlet and exit states gives the isentropic feed pump work,

$$\dot{w}_{x,12s} = \Delta h_{12s} = \int_1^{2s} \mathrm{d}h = \int_1^{2s} \frac{\mathrm{d}p}{\rho} \approx \frac{1}{\rho}\int_1^{2s} \mathrm{d}p = \frac{1}{\rho}\left(p_2 - p_1\right),$$

and using the definition of isentropic efficiency,

$$\dot{w}_{x,12} = \frac{\dot{w}_{x,12s}}{\eta_s} = \frac{p_2 - p_1}{\eta_s \rho}.$$

Reheat has three beneficial effects:

- Increases cycle efficiency, because the average temperature of heat addition increases.
- Increases specific work, because a greater area is enclosed on the T–s diagram.
- Reduces turbine wetness fraction, because the expansion is moved rightwards on the T–s diagram. This is a good thing because condensation of water droplets reduce the turbine efficiency and operating lifetime.

Q2: Steam turbine with reheat and feedheating

(a) Assuming the IP/LP expansion line is *straight*, the locus of states as the steam passes through the turbine is given by drawing a line between the inlet and exit states on the *h*–*s* chart in the Data Book. Where this line intersects the constant pressure line $p = 4\,\mathrm{bar}$ is the extraction state of one feedheater. The water inlet states are the feed pump outlets. The outlet states are saturated water at the local pressure. Treat each feedheater as an *adiabatic* mixing box; apply the **First Law** to balance the fluxes of enthalpy in and out. This gives two equations in the two unknown mass fractions.

(b) Again, this is Part I material. Be careful as arithmetic slips can be costly in Tripos.

(c) There are three important effects of feedheating:

- Efficiency increases, because the average temperature of heat addition increases. The heat required to raise the water to boiler inlet temperature does not come from outside the cycle; only external heat transfers crossing the engine control volume boundary are relevant here.
- Specific work reduces, because there is less steam yielding work in the turbine.
- Condenser heat flow reduces. This is important because the size and hence cost of the heat exchanger can be reduced.

Q3: Combustion in a steam turbine

(a) First convert the mass fractions f_i to mole fractions:

$$X_{\mathrm{C}} = \frac{f_{\mathrm{C}}/M_{\mathrm{C}}}{f_{\mathrm{C}}/M_{\mathrm{C}} + f_{\mathrm{H}}/M_{\mathrm{H}}} = 0.321, \qquad \text{and} \qquad X_{\mathrm{H}} = \frac{f_{\mathrm{H}}/M_{\mathrm{H}}}{f_{\mathrm{C}}/M_{\mathrm{C}} + f_{\mathrm{H}}/M_{\mathrm{H}}} = 0.679.$$

For stoichiometric combustion, each C requires one O_2, to make CO_2, and each H requires $\frac{1}{4}O_2$, to make $\frac{1}{2}H_2O$. So for one kilomole of fuel $N_{\mathrm{fuel}} = 1\,\mathrm{kmol}$ we require $N_{O_2} = X_{\mathrm{C}} + \frac{1}{4}X_H$ mole of O_2, and where $X_{O_2} = 0.21$ is the mole fraction of O_2 in air,

$$N_{\mathrm{air}} = \frac{X_{\mathrm{C}} + \frac{1}{4}X_H}{X_{O_2}} = 2.337\,\mathrm{kmol}.$$

Finally convert the molar air/fuel ratio back to mass and add 10% as requested,

$$\begin{aligned} M_{\mathrm{air}} &= 0.21(2\times 16) + 0.79(2\times 14) = 28.84\,\mathrm{kg\,kmol^{-1}} \\ M_{\mathrm{fuel}} &= 0.321(12) + 0.679(1) = 4.53\,\mathrm{kg\,kmol^{-1}} \\ AFR &= \frac{N_{\mathrm{air}}M_{\mathrm{air}}}{N_{\mathrm{fuel}}M_{\mathrm{fuel}}} \times 1.1 = 16.37 \end{aligned}$$

(b) The boiler efficiency is **defined** as follows, and given by the below expression as derived in the Lecture Notes,

$$\eta_{\mathrm{b}} = \frac{\text{actual heat addition}}{\text{fuel calorific value}} = \frac{\dot{Q}_{\mathrm{b}}}{\dot{m}_{\mathrm{f}} LCV} = 1 - \frac{(AFR+1)\, c_{p,\mathrm{prod}}\, (T_{\mathrm{X}} - T_0)}{LCV}.$$

Where $c_{p,\mathrm{prod}}$ is the specific heat capacity of the combustion products, T_0 is the temperature of the reactants, and T_{X} the stack temperature. It is given that the stack temperature is $\Delta T = 30\,^{\circ}\mathrm{C}$ greater than the temperature of the feed water entering the final feed pump, so, $T_{\mathrm{X}} = T_{\mathrm{sat}}(p = 40\,\mathrm{bar}) + \Delta T$. Then, the boiler efficiency can be found. The overall efficiency is **defined**,

$$\eta_{\mathrm{ov}} = \frac{\dot{W}_{x,\mathrm{net}}}{\dot{m}_{\mathrm{f}} LCV} = \frac{\dot{W}_{x,\mathrm{net}}}{\dot{Q}_{\mathrm{b}}} \times \frac{\dot{Q}_{\mathrm{b}}}{\dot{m}_{\mathrm{f}} LCV} = \eta_{\mathrm{cy}} \times \eta_{\mathrm{b}}.$$

(c) Heat exchanger effectiveness is **defined**,

$$\varepsilon = \frac{\text{actual heat transfer}}{\text{maximum heat transfer}} = \frac{(\dot{m} c_p \Delta T)_{\mathrm{hot}}}{(\dot{m} c_p)_{\min} (T_{\max} - T_{\min})} = \frac{(\dot{m} c_p \Delta T)_{\mathrm{cold}}}{(\dot{m} c_p)_{\min} (T_{\max} - T_{\min})},$$

where the actual heat flow $\dot{m} c_p \Delta T$ is the same for both hot and cold sides because the heat exchanger is assumed *adiabatic*. The combustion products have a higher specific heat capacity, and the same mass flow rate, so the equation simplifies using the cold side,

$$\varepsilon = \frac{T_1 - T_0}{T_2 - T_0}, \tag{1}$$

where states 0, 1, and 2 are: preheater inlet (atmospheric), boiler inlet, and boiler outlet. Applying the **First Law** to the preheater,

$$AFR c_{p,\mathrm{air}} (T_1 - T_0) = (AFR+1) c_{p,\mathrm{prod}} (T_2 - T_{\mathrm{X}}). \tag{2}$$

Equations (1) and (2) can be solved simultaneously to yield the new T_{X}, and the same analysis as in part (b) gives η_{b} and η_{ov}.

Q4: Combined cycle analysis

(a) Assuming the HRSG is *adiabatic*, applying the **First Law** between the gas turbine outlet and the pinch point, all states are known. This fixes the ratio of steam to gas mass flow rates (with a factor of $c_{p,\mathrm{prod}}$ which is not yet known). Now applying the **First Law** again to the entire HRSG allows calculation of the stack temperature. The HRSG efficiency, analogous to η_{b}, is given by,

$$\eta_{\mathrm{b}} = \frac{\text{actual heat transfer}}{\text{maximum heat transfer}} = \frac{\text{enthalpy drop in HRSG}}{\text{enthalpy drop to environment}},$$

that is, the maximum possible heat transfer is when the gas turbine exhaust is cooled all the way down to the environment temperature. The rest of the question is relating to the ‘General Thermodynamic Analysis’ from the Lecture Notes.

(b) The point of the preheating loop is to avoid putting very cold water into the HRSG. If we do this, the temperature of the water pipes will drop allowing condensation of flue gases on the outside of the pipes. This is a local effect, in that a high bulk temperature of the flue gas is not sufficient to prevent it. The thermodynamic states we have to consider are as follows,

- Water entering the drum (top left on the diagram) is wet-saturated at the condenser pressure, *neglecting feed pump work*.
- Water leaving the drum (bottom left and right on the diagram) is wet-saturated at the loop pressure.
- The steam re-entering the drum (top right on diagram) is dry-saturated at the loop pressure, *as stated* in the question.

Taking the drum as *adiabatic* and in *steady-state*, applying the **First Law** gives the diverted mass flow fraction which is compatible with the three states. The stack temperature is unchanged (draw diagrams and control volumes for the new and old arrangements to see why). The preheating loop works because the whole system is isothermal at the saturation temperature set by the loop pressure, $T_{\text{sat}}(p = 1\,\text{bar})$. However, as we are effectively using water we have already heated to preheat the incoming water, there will be a start up transient where this is not the case and the condensation risk remains.

(c) We take the flue gas down to a new pinch point with a difference of $\Delta T = 10\,^{\circ}\text{C}$,

$$T_{\text{X,new}} = T_{\text{sat}}(p = 1\,\text{bar}) + \Delta T$$

so the extra heat transferred is,

$$\dot{Q}_{\text{extra}} = \dot{m}_1 c_{p,\text{prod}}(T_{\text{X,new}} - T_{\text{X,old}}).$$

Some manipulations with the steam power output, efficiency and other quantities from part (a) allow $\dot{m}_1 c_{p,\text{prod}}$ to be eliminated and $\dot{Q}_{\text{extra}}$ to be evaluated. The extra power is simply, $\dot{W}_{\text{extra}} = \eta_{\text{extra}}\dot{Q}_{\text{extra}}$, i.e. it is not expected that the steam cycle should be calculated again, just use the prescribed value of efficiency. The gas turbine remains unchanged, so the heat input is the same, allowing some manipulation to get the new combined cycle efficiency.

Q5: Optimising simple-cycle pressure ratio

(a) The basic idea is to find an expression for the specific work, non-dimensionalise and rearrange in terms of the desired quantities, and then differentiate with respect to the isentropic temperature ratio to find the maximum location.

(b) There is a slightly more complicated expression to differentiate here, but the method remains the same. Specifically, we find,

$$\eta_{\text{gt}}(\tau) = \frac{\frac{1}{\eta_{\text{c}}}(1-\tau) + \eta_{\text{t}}\theta(1-\frac{1}{\tau})}{\theta - \frac{1}{\eta_{\text{c}}}(1-\tau)}.$$

It is easiest to 'cross-multiply' and differentiate implicitly,

$$\left[\theta - \frac{1}{\eta_\mathrm{c}}(1-\tau)\right]\eta_\mathrm{gt}(\tau) = \frac{1}{\eta_\mathrm{c}}(1-\tau) + \eta_\mathrm{t}\theta(1-\frac{1}{\tau})$$

$$\left[\theta - \frac{1}{\eta_\mathrm{c}}(1-\tau)\right]\eta'_\mathrm{gt}(\tau) + \frac{1}{\eta_\mathrm{c}}\eta_\mathrm{gt}(\tau) = -\frac{1}{\eta_\mathrm{c}} + \eta_\mathrm{t}\theta\frac{1}{\tau^2}$$

Then setting $\eta'_\mathrm{gt}(\tau = \tau_\eta) = 0$ and rearranging gives the desired result.

(c) Recall that the isentropic temperature ratio is actually a pressure ratio with an extra exponent to simplify notation,

$$\tau = PR^{\frac{\gamma-1}{\gamma}} \qquad \Rightarrow \qquad PR = \tau^{\frac{\gamma}{\gamma-1}}.$$

We want to show that $0.3PR_\eta \leq PR_w \leq 0.5PR_\eta$ so work out the ratio,

$$\left(\frac{PR_w}{PR_\eta}\right)_\mathrm{sc} = \frac{\tau_w^{\frac{\gamma}{\gamma-1}}}{\tau_\eta^{\frac{\gamma}{\gamma-1}}} = \left(\frac{\tau_w}{\tau_\eta}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{\eta_c\eta_t\theta}{\frac{\eta_c\eta_t\theta}{1-\eta^*_\mathrm{gt}}}\right)^{\frac{1}{2}\frac{\gamma}{\gamma-1}} = \left(1-\eta^*_\mathrm{gt}\right)^{\frac{1}{2}\frac{\gamma}{\gamma-1}}$$

A typical gas turbine in simple cycle has $\eta^*_\mathrm{gt} \approx 40\%$, in which case the ratio evaluates to $PR_w/PR_\eta \approx 0.41$, in the middle of the suggested bounds.

Q6: Optimising combined-cycle pressure ratio

(a) This is a similar analysis to Q4(a) to find the steam/gas mass flow ratio. Note that changing the pressure only changes β and the gas turbine exhaust temperature. The pinch-point temperature is fixed by the steam cycle, atmospheric conditions are constant, and the turbine inlet temperature is set by θ.

(b) Again, similar to previous questions, just more long-winded algebra.

(c) It is strongly recommended that you use the 'cross-multiplying' technique illustrated in Q5(b). Otherwise the algebra is very lengthy.

(d) Proceeding as in Q5(c),

$$\left(\frac{PR_w}{PR_\eta}\right)_\mathrm{cc} = \left(\frac{\eta_c\eta_t\theta\left(1-\frac{\eta_\mathrm{st}}{\alpha}\right)}{\frac{\eta_c\eta_t\theta\left(1-\frac{\eta_\mathrm{st}}{\alpha}\right)}{1-\eta^*_\mathrm{cc}}}\right)^{\frac{1}{2}\frac{\gamma}{\gamma-1}} = (1-\eta^*_\mathrm{cc})^{\frac{1}{2}\frac{\gamma}{\gamma-1}},$$

which with $\eta^*_\mathrm{cc} \approx 60\%$ evaluates to $(PR_w/PR_\eta)_\mathrm{cc} \approx 0.2$, a much lower value than for the simple cycle case. In practice, the same design of gas turbine might be used in a combined cycle mode for power generation applications, where efficiency is most important, and in a simple cycle mode for industrial applications, e.g. on an oil rig where fuel is free and power output is the only concern. For this situation,

$$\frac{PR_{w,\mathrm{sc}}}{PR_{\eta,\mathrm{cc}}} = \left(\frac{\eta_c \eta_t \theta}{\frac{\eta_c \eta_t \theta \left(1 - \frac{\eta_{\mathrm{st}}}{\alpha}\right)}{1 - \eta_{\mathrm{cc}}^*}} \right)^{\frac{1}{2}\frac{\gamma}{\gamma-1}} = \left(\frac{1 - \eta_{\mathrm{cc}}^*}{1 - \frac{\eta_{\mathrm{st}}}{\alpha}} \right)^{\frac{1}{2}\frac{\gamma}{\gamma-1}} .$$

Taking $\eta_{\mathrm{cc}}^* \approx 60\%$, $\alpha \approx 0.6$, and $\eta_{\mathrm{st}} \approx 35\%$ this evaluates to $PR_{w,\mathrm{sc}}/PR_{\eta,\mathrm{cc}} \approx 0.96$, showing that the same pressure ratio gives good performance in both situations.