IB Paper 4: Fluid Dynamics
Hints on Examples Paper 3

James Brind
jb753@cam.ac.uk

Michaelmas Term 2019

1. According to the given expression, mean free path is a function of number of molecules per unit volume, which we need to relate to pressure somehow (see Question S.2). If we suck air out of the pump and reduce the pressure, eventually the mean free path increases to be of the same order as the pump diameter. There are very few molecules left in the pump — think about the implications for our continuum assumption, see Lectures §1.2.

S2. An equation of state relates the thermodynamic state variables of a particular fluid, for example of the form $\rho = \mathrm{fn}(p, T)$. Two thermodynamic properties and an equation of state are sufficient to calculate all other thermodynamic properties. The ideal gas law, often a good approximation, is $\rho(p, T) = p/RT$. An incompressible fluid has a simpler equation of state. Complex fluids such as steam or refrigerants do not have nice analytic equations of state, and instead Engineers rely on look-up tables, where ρ has been measured at many combinations of p and T.

S4. The new operator is the material derivative $\mathrm{D}/\mathrm{D}t$, defined in Lectures §3.1. Both $\mathrm{d}/\mathrm{d}t$ and $\mathrm{D}/\mathrm{D}t$ are derivatives following a fluid particle; their distinction is rather subtle. $\mathrm{d}/\mathrm{d}t$ follows a specific particle, and is defined for points on a streamline. If the identity of the particle is not of concern, then $\mathrm{D}/\mathrm{D}t$ is defined at any point in space and time for all particles passing through that point.

3. We want to know if mass is conserved *everywhere* in the flow. A control volume analysis cannot show this, so we use vector calculus. From the Thermofluids Databook, we need to check if $\nabla \cdot (\rho \mathbf{V}) = 0$. Similarly, to prove that vorticity is zero *throughout* the flow, we need to show $\omega = \nabla \times \mathbf{V} = 0$. We are given an expression for $\mathbf{V}$, and definitions for ∇ are in the Maths Databook. If the flow has no vorticity, some useful assumptions can be made, Lectures §3.3, yielding the static pressure.

4. First, consider conservation of mass through the channel and how the average velocity must vary. Next, think of the curvature of a streamline, and what this means for the pressure on upper and lower walls. Finally, identify the inflection points in the streamline where curvature changes direction. This is sufficient to make the sketch.

5. The key for part (b) is to take a dot product of the Euler equation with velocity, as this yields the component of each term that is parallel to the streamline (always tangential to the velocity).

6. Flows with no vorticity are easier to analyse, as we found in Question 3. This gives us an equation that can be manipulated to form the first expression. You should recognise the second expression.

7., 8. These are very similar to the Couette and Poiseuille flow examples in the Lectures §4.4 and §4.5 respectively. The only complication is finding a flow rate from a known velocity profile by integration. To simplify the Navier–Stokes equations, we make the following assumptions: two-dimensional, steady flow with parallel streamlines. We can show from continuity that $\mathbf{V}$ is not a function of x.

9. This is combined Couette and Poiseuille flow, Lectures §4.7. Consider the difference in height between the two sides, and how the velocity profiles must then differ if mass is to be conserved. You might find the results from Question 8. useful for part (b).

10., 11. More force balances. You need to be well-practiced at these as it is easy to slip up on the algebra. We also need to be careful because in a cylindrical coordinate system the areas on the outer and inner faces are different, Lectures §4.9. Don't forget to include gravity where needed.